1 Draw diagrams, as in Examples **1** and **2**, to show the following angles. Mark in the acute angle that OP makes with the x-axis.

a $-80°$ **b** $100°$ **c** $200°$

d $165°$ **e** $-145°$ **f** $225°$

g $280°$ **h** $330°$ **i** $-160°$

j $-280°$ **k** $\frac{3\pi}{4}$ **l** $\frac{7\pi}{6}$

m $-\frac{5\pi}{3}$ **n** $-\frac{5\pi}{8}$ **o** $\frac{19\pi}{9}$

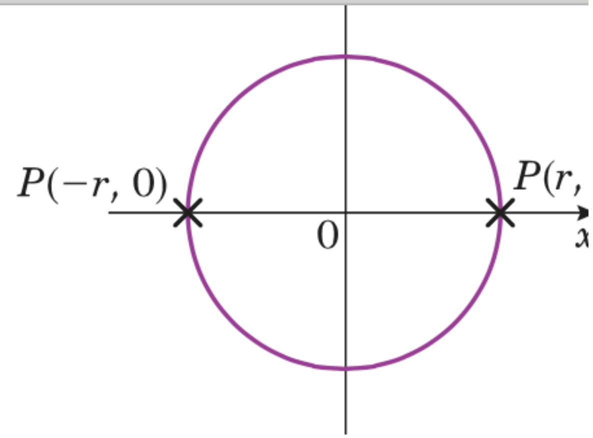
2 State the quadrant that OP lies in when the angle that OP makes with the positive x-axis is:

a $400°$ **b** $115°$ **c** $-210°$

d $255°$ **e** $-100°$ **f** $\frac{7\pi}{8}$

g $-\frac{11\pi}{6}$ **h** $\frac{13\pi}{7}$

When $y = 0$, $\tan\theta = 0$. This is when P is at $(r, 0)$ or $(-r, 0)$.



- Tan $\theta = 0$ when θ is 0° or an even multiple of 90° $\left(\text{or } \frac{\pi}{2} \text{ radians}\right)$.

Exercise 8B

(*Note*: do not use a calculator.)

1 Write down the values of:

a $\sin(-90)°$ **b** $\sin 450°$ **c** $\sin 540°$ **d** $\sin(-450)°$

e $\cos(-180)°$ **f** $\cos(-270)°$ **g** $\cos 270°$ **h** $\cos 810°$

i $\tan 360°$ **j** $\tan(-180)°$

2 Write down the values of the following, where the angles are in radians:

a $\sin\frac{3\pi}{2}$ **b** $\sin\left(-\frac{\pi}{2}\right)$ **c** $\sin 3\pi$ **d** $\sin\frac{7\pi}{2}$

e $\cos 0$ **f** $\cos\pi$ **g** $\cos\frac{3\pi}{2}$ **h** $\cos\left(-\frac{3\pi}{2}\right)$

i $\tan\pi$ **j** $\tan(-2\pi)$

Exercise 8C

(*Note*: Do not use a calculator.)

1 By drawing diagrams, as in Example **6**, express the following in terms of trigonometric ratios of acute angles:

a $\sin 240°$ **b** $\sin(-80)°$ **c** $\sin(-200)°$ **d** $\sin 300°$ **e** $\sin 460°$

f $\cos 110°$ **g** $\cos 260°$ **h** $\cos(-50)°$ **i** $\cos(-200)°$ **j** $\cos 545°$

k $\tan 100°$ **l** $\tan 325°$ **m** $\tan(-30)°$ **n** $\tan(-175)°$ **o** $\tan 600°$

p $\sin\frac{7\pi}{6}$ **q** $\cos\frac{4\pi}{3}$ **r** $\cos\left(-\frac{3\pi}{4}\right)$ **s** $\tan\frac{7\pi}{5}$ **t** $\tan\left(-\frac{\pi}{3}\right)$

u $\sin\frac{15\pi}{16}$ **v** $\cos\frac{8\pi}{5}$ **w** $\sin\left(-\frac{6\pi}{7}\right)$ **x** $\tan\frac{15\pi}{8}$

2 Given that θ is an acute angle measured in degrees, express in terms of $\sin\theta$:

a $\sin(-\theta)$ **b** $\sin(180° + \theta)$ **c** $\sin(360° - \theta)$

d $\sin -(180° + \theta)$ **e** $\sin(-180° + \theta)$ **f** $\sin(-360° + \theta)$

g $\sin(540° + \theta)$ **h** $\sin(720° - \theta)$ **i** $\sin(\theta + 720°)$

3 Given that θ is an acute angle measured in degrees, express in terms of $\cos\theta$ or $\tan\theta$:

a $\cos(180° - \theta)$ **b** $\cos(180° + \theta)$ **c** $\cos(-\theta)$

d $\cos -(180° - \theta)$ **e** $\cos(\theta - 360°)$ **f** $\cos(\theta - 540°)$

g $\tan(-\theta)$ **h** $\tan(180° - \theta)$ **i** $\tan(180° + \theta)$

j $\tan(-180° + \theta)$ **k** $\tan(540° - \theta)$ **l** $\tan(\theta - 360°)$

The results obtained in questions **2** and **3** are true for all values of θ.

3 **a** Use your calculator to evaluate: **i** $\frac{1}{\sqrt{2}}$ **ii** $\frac{\sqrt{3}}{2}$

b Copy and complete the following table. Use your calculator to evaluate the trigonometric ratios, then **a** to write them exactly.

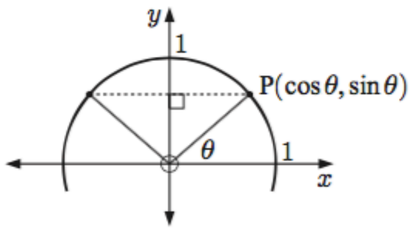
θ (degrees)	$30°$	$45°$	$60°$	$135°$	$150°$	$240°$	$315°$
θ (radians)							
sine							
cosine							
tangent							

4 **a** Use your calculator to evaluate:

i $\sin 100°$ **ii** $\sin 80°$ **iii** $\sin 120°$ **iv** $\sin 60°$
v $\sin 150°$ **vi** $\sin 30°$ **vii** $\sin 45°$ **viii** $\sin 135°$

b Use the results from **a** to copy and complete:
$\sin(180° - \theta) = \ldots\ldots$

c Justify your answer using the diagram alongside:



d Find the obtuse angle with the same sine as:

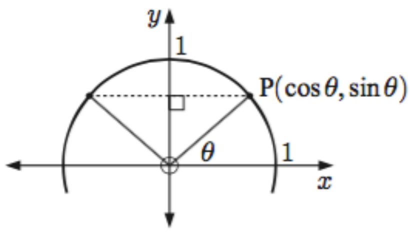
i $45°$ **ii** $51°$ **iii** $\frac{\pi}{3}$ **iv** $\frac{\pi}{6}$

5 **a** Use your calculator to evaluate:

i $\cos 70°$ **ii** $\cos 110°$ **iii** $\cos 60°$ **iv** $\cos 120°$
v $\cos 25°$ **vi** $\cos 155°$ **vii** $\cos 80°$ **viii** $\cos 100°$

b Use the results from **a** to copy and complete:
$\cos(180° - \theta) = \ldots\ldots$

c Justify your answer using the diagram alongside:



d Find the obtuse angle which has the negative cosine of:

i $40°$ **ii** $19°$ **iii** $\frac{\pi}{5}$ **iv** $\frac{2\pi}{5}$

6 Without using your calculator, find:

a $\sin 137°$ if $\sin 43° \approx 0.6820$
b $\sin 59°$ if $\sin 121° \approx 0.8572$
c $\cos 143°$ if $\cos 37° \approx 0.7986$
d $\cos 24°$ if $\cos 156° \approx -0.9135$
e $\sin 115°$ if $\sin 65° \approx 0.9063$
f $\cos 132°$ if $\cos 48° \approx 0.6691$

7 **a** Copy and complete:

Quadrant	*Degree measure*	*Radian measure*	$\cos\theta$	$\sin\theta$	$\tan\theta$
1	$0° < \theta < 90°$	$0 < \theta < \frac{\pi}{2}$	positive	positive	
2					
3					
4					

EXERCISE 8D.2

1 Find two angles θ on the unit circle, with $0 \leqslant \theta \leqslant 2\pi$, such that:

a $\tan\theta = 4$ **b** $\cos\theta = 0.83$ **c** $\sin\theta = \frac{3}{5}$

d $\cos\theta = 0$ **e** $\tan\theta = 1.2$ **f** $\cos\theta = 0.7816$

g $\sin\theta = \frac{1}{11}$ **h** $\tan\theta = 20.2$ **i** $\sin\theta = \frac{39}{40}$

2 Find two angles θ on the unit circle, with $0 \leqslant \theta \leqslant 2\pi$, such that:

a $\cos\theta = -\frac{1}{4}$ **b** $\sin\theta = 0$ **c** $\tan\theta = -3.1$

d $\sin\theta = -0.421$ **e** $\tan\theta = -6.67$ **f** $\cos\theta = -\frac{2}{17}$

g $\tan\theta = -\sqrt{5}$ **h** $\cos\theta = \frac{-1}{\sqrt{3}}$ **i** $\sin\theta = -\frac{\sqrt{2}}{\sqrt{5}}$

EXERCISE 8E

1 Use a unit circle diagram to find exact values for $\sin\theta$, $\cos\theta$, and $\tan\theta$ for θ equal to:

a $\frac{\pi}{4}$ **b** $\frac{3\pi}{4}$ **c** $\frac{7\pi}{4}$ **d** π **e** $\frac{-3\pi}{4}$

2 Use a unit circle diagram to find exact values for $\sin\beta$, $\cos\beta$, and $\tan\beta$ for β equal to:

a $\frac{\pi}{6}$ **b** $\frac{2\pi}{3}$ **c** $\frac{7\pi}{6}$ **d** $\frac{5\pi}{3}$ **e** $\frac{11\pi}{6}$

3 Find the exact values of:

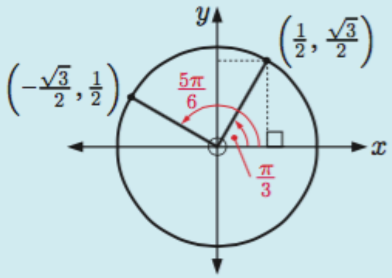
a $\cos 120°$, $\sin 120°$, and $\tan 120°$ **b** $\cos(-45°)$, $\sin(-45°)$, and $\tan(-45°)$

4 **a** Find the exact values of $\cos 90°$ and $\sin 90°$.

b What can you say about $\tan 90°$?

Example 15 **Self Tutor**

Without using a calculator, show that $8\sin(\frac{\pi}{3})\cos(\frac{5\pi}{6}) = -6$.



$\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ and $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$

$\therefore\ 8\sin(\frac{\pi}{3})\cos(\frac{5\pi}{6}) = 8(\frac{\sqrt{3}}{2})(-\frac{\sqrt{3}}{2})$

$= 2(-3)$

$= -6$

5 Without using a calculator, evaluate:

a $\sin^2 60°$ **b** $\sin 30°\cos 60°$ **c** $4\sin 60°\cos 30°$

d $1-\cos^2(\frac{\pi}{6})$ **e** $\sin^2(\frac{2\pi}{3})-1$ **f** $\cos^2(\frac{\pi}{4})-\sin(\frac{7\pi}{6})$

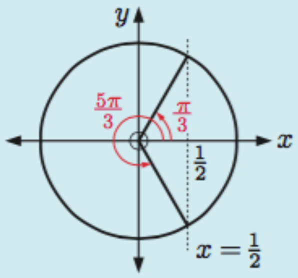
g $\sin(\frac{3\pi}{4})-\cos(\frac{5\pi}{4})$ **h** $1-2\sin^2(\frac{7\pi}{6})$ **i** $\cos^2(\frac{5\pi}{6})-\sin^2(\frac{5\pi}{6})$

j $\tan^2(\frac{\pi}{3})-2\sin^2(\frac{\pi}{4})$ **k** $2\tan(-\frac{5\pi}{4})-\sin(\frac{3\pi}{2})$ **l** $\frac{2\tan 150°}{1-\tan^2 150°}$

Check all answers using your calculator.

Example 16 **Self Tutor**

Find all angles $0 \leqslant \theta \leqslant 2\pi$ with a cosine of $\frac{1}{2}$.



Since the cosine is $\frac{1}{2}$, we draw the vertical line $x = \frac{1}{2}$.

Because $\frac{1}{2}$ is involved, we know the required angles are multiples of $\frac{\pi}{6}$.

They are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

6 Find all angles between $0°$ and $360°$ with:

a a sine of $\frac{1}{2}$ **b** a sine of $\frac{\sqrt{3}}{2}$ **c** a cosine of $\frac{1}{\sqrt{2}}$

d a cosine of $-\frac{1}{2}$ **e** a cosine of $-\frac{1}{\sqrt{2}}$ **f** a sine of $-\frac{\sqrt{3}}{2}$

7 Find all angles between 0 and 2π (inclusive) which have:

a a tangent of 1 **b** a tangent of -1 **c** a tangent of $\sqrt{3}$

d a tangent of 0 **e** a tangent of $\frac{1}{\sqrt{3}}$ **f** a tangent of $-\sqrt{3}$

8 Find all angles between 0 and 4π with:

a a cosine of $\frac{\sqrt{3}}{2}$ **b** a sine of $-\frac{1}{2}$ **c** a sine of -1

9 Find θ if $0 \leqslant \theta \leqslant 2\pi$ and:

a $\cos\theta = \frac{1}{2}$ **b** $\sin\theta = \frac{\sqrt{3}}{2}$ **c** $\cos\theta = -1$ **d** $\sin\theta = 1$

e $\cos\theta = -\frac{1}{\sqrt{2}}$ **f** $\sin^2\theta = 1$ **g** $\cos^2\theta = 1$ **h** $\cos^2\theta = \frac{1}{2}$

i $\tan\theta = -\frac{1}{\sqrt{3}}$ **j** $\tan^2\theta = 3$

10 Find *all* values of θ for which $\tan\theta$ is: **a** zero **b** undefined.

Then $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ and $\tan 45° = 1$

Exercise 8D

1 Express the following as trigonometric ratios of either 30°, 45° or 60°, and hence find their exact values.

a $\sin 135°$ **b** $\sin(-60°)$ **c** $\sin 330°$ **d** $\sin 420°$ **e** $\sin(-300°)$

f $\cos 120°$ **g** $\cos 300°$ **h** $\cos 225°$ **i** $\cos(-210°)$ **j** $\cos 495°$

k $\tan 135°$ **l** $\tan(-225°)$ **m** $\tan 210°$ **n** $\tan 300°$ **o** $\tan(-120°)$

2 In Section 8.3 you saw that $\sin 30° = \cos 60°$, $\cos 30° = \sin 60°$, and $\tan 60° = \frac{1}{\tan 30°}$.

These are particular examples of the general results: $\sin(90° - \theta) = \cos\theta$, and

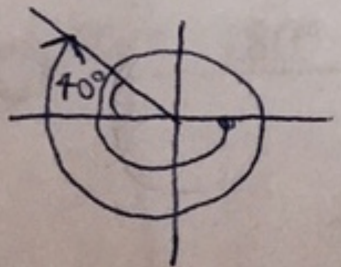
$\cos(90° - \theta) = \sin\theta$, and $\tan(90° - \theta) = \frac{1}{\tan\theta}$, where the angle θ is measured in degrees.

Use a right-angled triangle ABC to verify these results for the case when θ is acute.

Find one <u>POSITIVE</u> and one <u>NEGATIVE</u> co-terminal angle from $0 \leq \theta \leq 360$ for each given degree. Draw the angles. (6 pts)

14) $-580°$

180
540

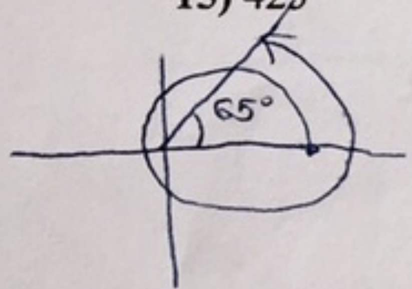


Positive:

Negative:

140°, -220°

15) $425°$

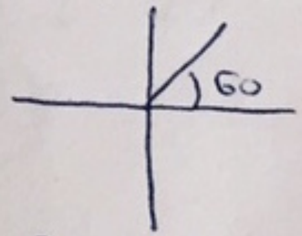


Positive:

Negative:

65°, -295°

coterminal angles



eg 60°, -300, 7
420

BONUS:

What is the 6th planet away from our sun?

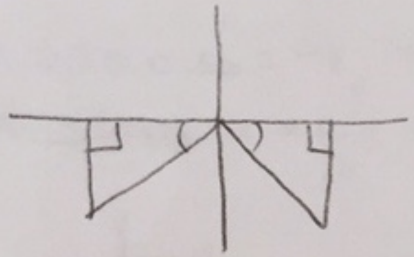
Saturn!

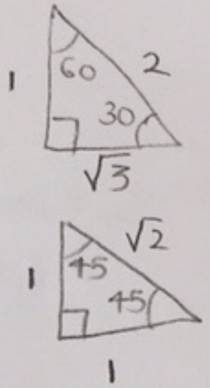
limits

Find all degree(s) and its radian measures. Draw the triangles. (4 pts) haven't been given limits?

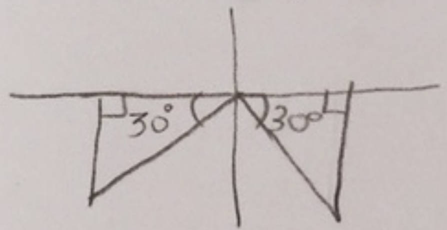
3) Angles whose sine is $-\frac{1}{2}$

$\sin x = -\frac{1}{2}$





$\sin^{-1}(\frac{1}{2})$



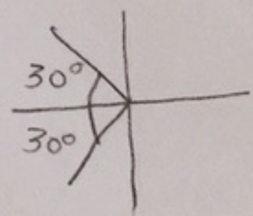
210°, 330°, ...

Proper answer: $-30 + 360n$
$210 + 360n$

4) Angles whose cosine is $-\frac{\sqrt{3}}{2}$

$\cos x = -\frac{\sqrt{3}}{2}$

$\cos^{-1}(\frac{\sqrt{3}}{2})$



150°, 210° ...

Proper Answer: $\pm 150 + 360n$
$\pm 210 + 360n$

Graph one full cycle for each function. Make graph large enough for me to see. Label both axes. (18 pts).

C

9) $y = 4\tan(3\theta)$ Amp A = 4 Period? T = $\frac{2\pi}{3}$ vertical shift V.S.= 0 Phase shift P.S. = 0

$P = \frac{2\pi}{6}$

$P = \frac{2\pi}{3}$

$2\theta - \frac{\pi}{2}$

10) $y = -2\sin 2(\theta - \frac{\pi}{4}) - 3$ A = 2 T = $\frac{2\pi}{2} = \pi$ V.S. = −3 P.S. = $\frac{\pi}{2}$

C

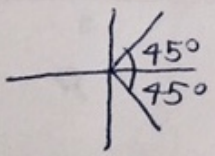
Solve each equation for θ in radians from $0 \le \theta \le 2\pi$. Draw the triangles. (12 pts)

5) $2\cos\theta - \sqrt{2} = 0$

$\cos\theta = \frac{\sqrt{2}}{2}$

ref angle $= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$= 45°$

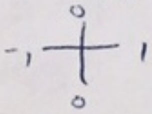


45°, 315°

6) $2\cos\theta\sin\theta + \cos\theta = 0$

$\cos\theta(2\sin\theta + 1) = 0$

$\cos\theta = 0$, $\sin\theta = -\frac{1}{2}$



$\frac{\pi}{2}, \frac{3\pi}{2}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7) $2\csc^2\theta - 2\csc\theta = 4$

let $y = \text{cosec}$

$2y^2 - 2y - 4 = 0$

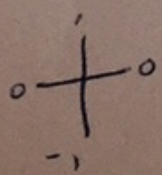
$y^2 - y - 2 = 0$

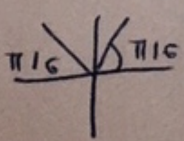
$(y+1)(y-2) = 0$

$y = -1, 2$

$\text{cosec}\,\theta = -1$, $\text{cosec}\,\theta = 2$

$\Rightarrow \sin\theta = -1$, $\sin\theta = \frac{1}{2}$





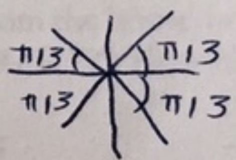
$\theta = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

8) $4\cos^2\theta - 1 = 0$

$4\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$



$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$